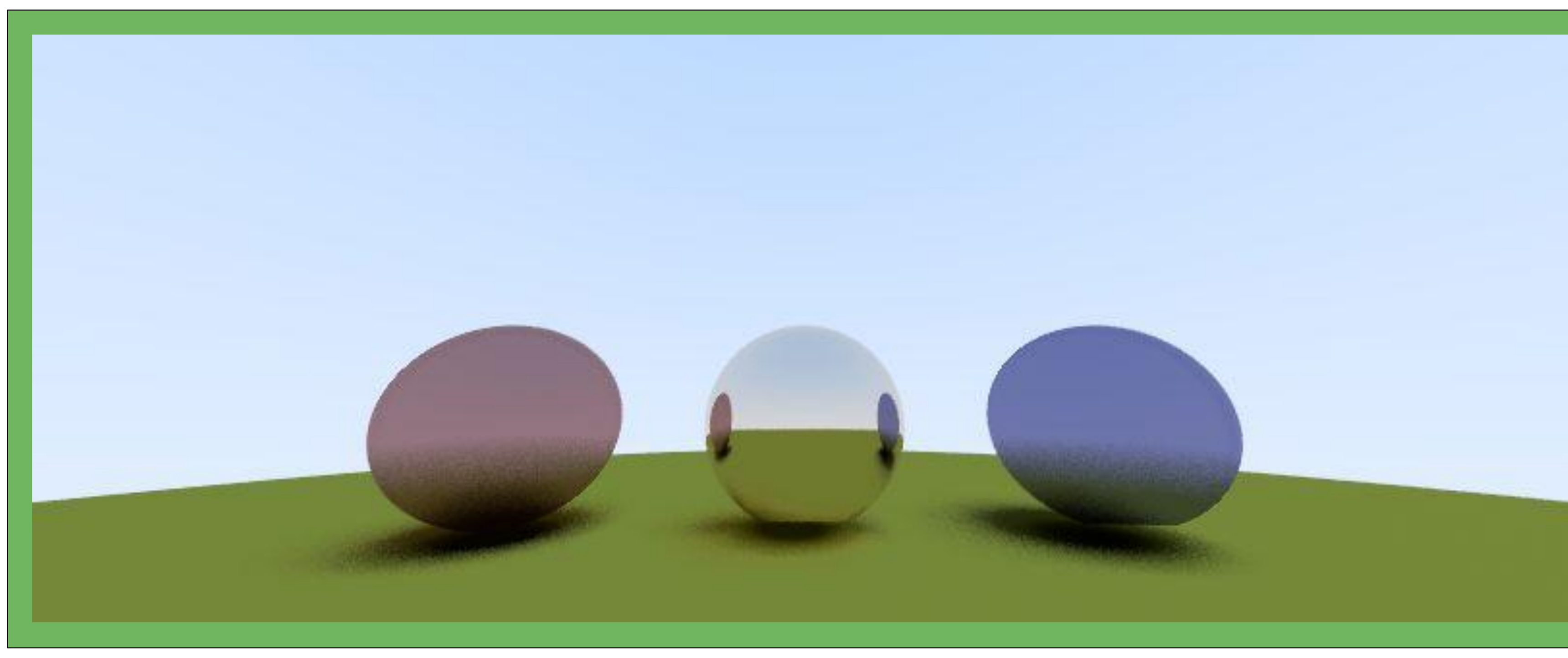
Numerical Mathematical Methods and their Applications

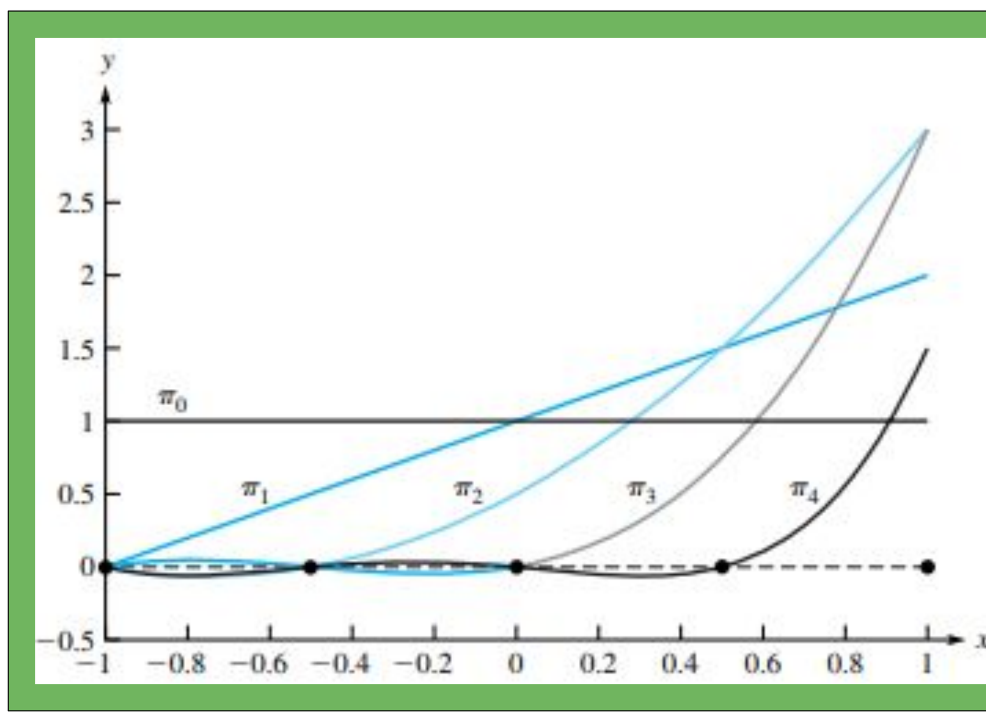
By Elliot Foley

Numerical Mathematical Methods and their Applications is a big title you probably haven't heard. However, we can break it down into its parts in order to better understand it. This is exactly what we are doing in the field! We take a big problem, and break it down into little pieces that a computer can handle. For the following applications, we display only a few approaches to demonstrate the capabilities of numerical computing.



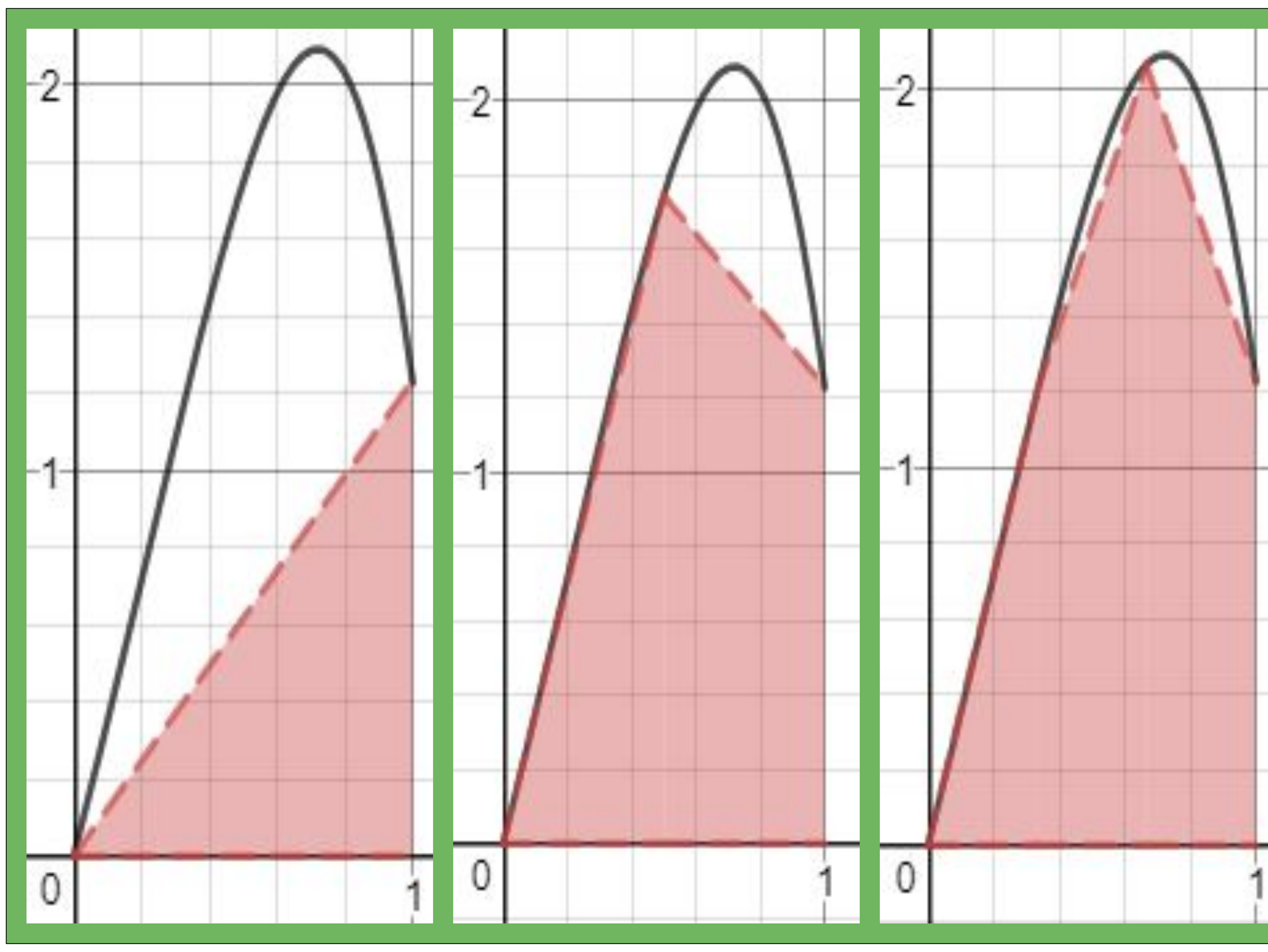
Ray Tracing

Simulating the process of vision. This is how animated films are made.



By creating Interpolating Polynomials, we can estimate differentiation of unknown functions by the derivatives of the polynomials.

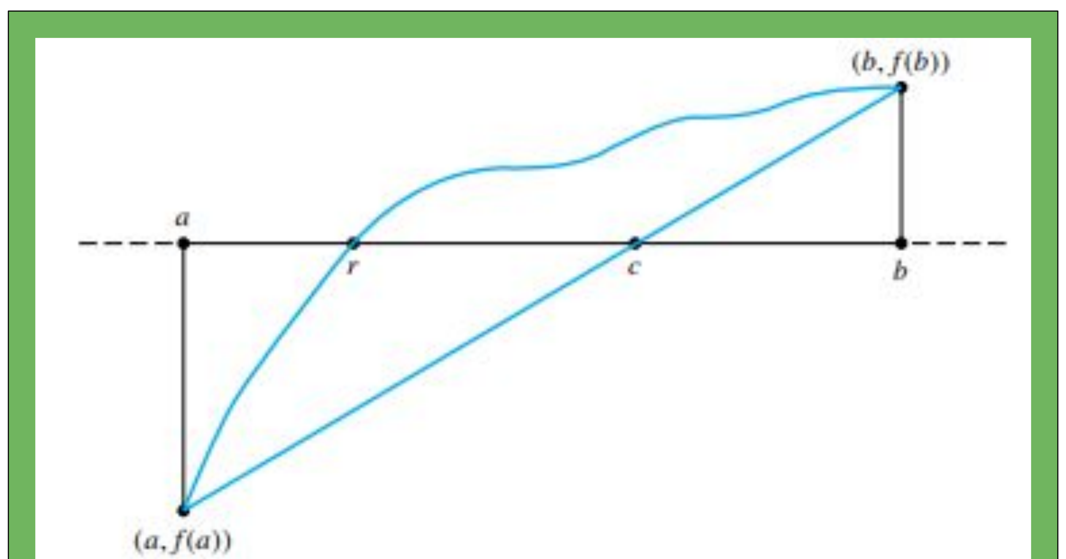
Interpolation and Numerical Differentiation



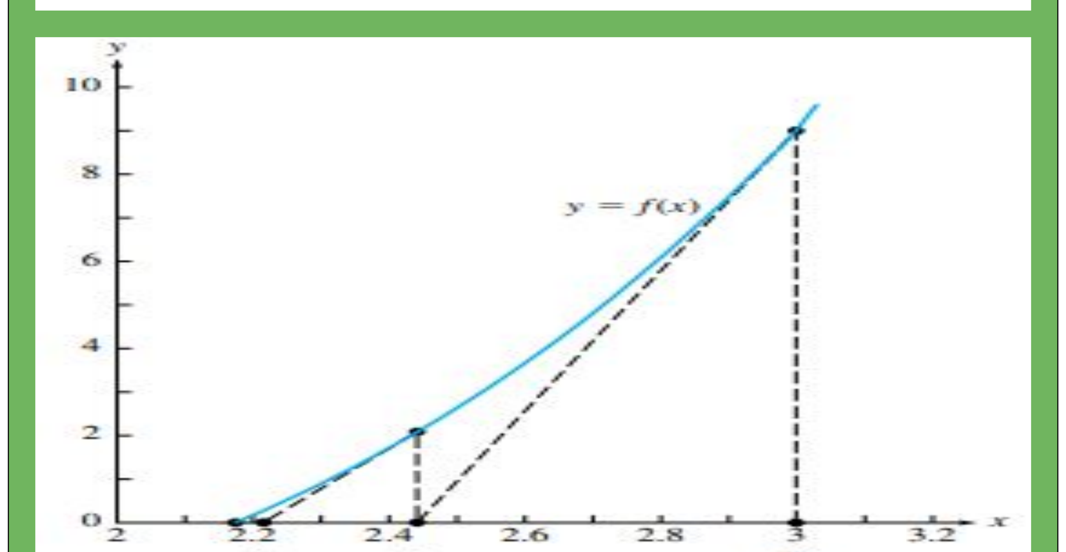
Numerical Integration

Using the Composite Trapezoid Rule

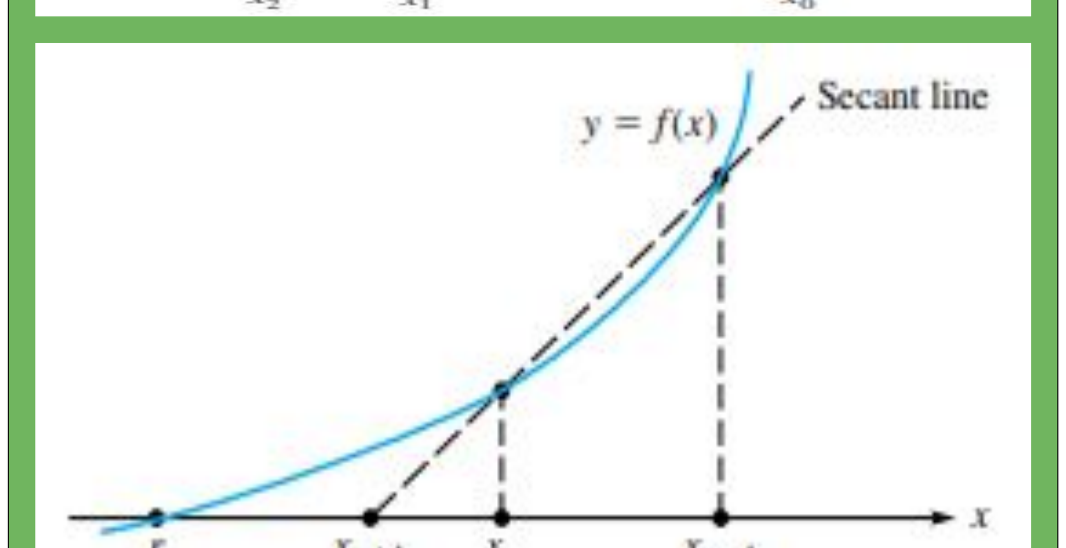
There are other methods, such as rectangles, parabolas, and higher-order polynomials, but we only show trapezoids here.



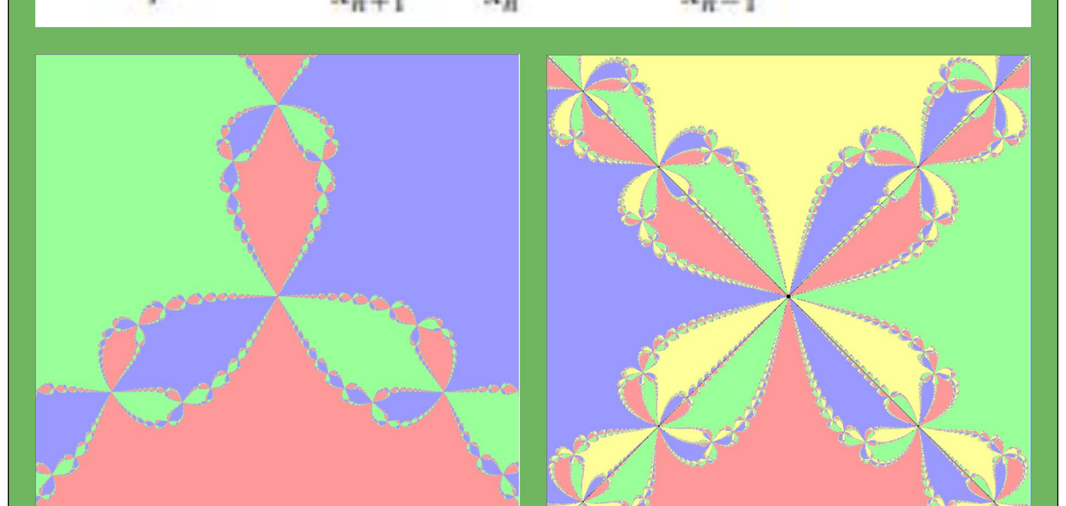
The Bisection Method heavily utilizes the Intermediate Value Theorem of Calculus.



Newton's Method takes the derivative of the function at a point into account to create a better estimate of the root.

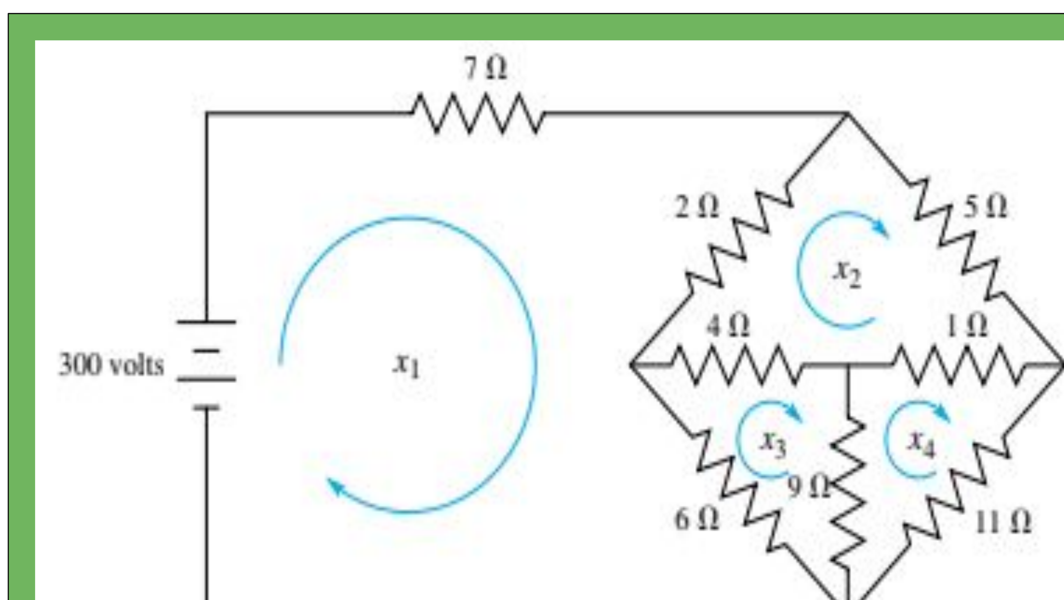


The Secant Method is almost as fast as Newton's, but does not take the derivative.



These are maps of the roots of polynomials y=x^3-1 and y=x^4-1 on the complex plane.

Locating Roots of Equations



$$\begin{cases} 15x_1 - 2x_2 - 6x_3 \qquad = 300 \\ -2x_1 + 12x_2 - 4x_3 - x_4 = 0 \\ -6x_1 - 4x_2 + 19x_3 - 9x_4 = 0 \\ \qquad - x_2 - 9x_3 + 21x_4 = 0 \end{cases}$$

Systems of Linear Equations

You have probably heard the buzzwords "Artificial Intelligence" and "Machine Learning." However, they are procedures, just like these, and not all that much more complex. They follow the same pattern of being able to be broken down into small parts.

References: "Numerical Mathematics and Computing by" Ward Cheney and David Kincaid

Acknowledgements:

Prof. Eugene Zhang, Prof. Yue Zhang,
Jinta Zheng,
Fariba Khan,
and to Saturday Academy for providing this learning opportunity.